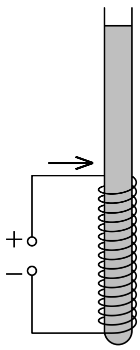
1 Three balls

Three small identical balls (denoted as A, B, and C) of mass m each are connected with two massless rods of length ℓ so that one of the rods connects the balls A and B, and the other rod connects the balls B and C. The connection at the ball B is hinged, and the angle between the rods can change effortlessly. The system rests in weightlessness so that all the balls lie on one line. The ball A is given instantaneously a velocity perpendicular to the rods.

Find the minimal distance d between the balls A and C during the subsequent motion of the system. Any friction is to be neglected.

2 Solenoid



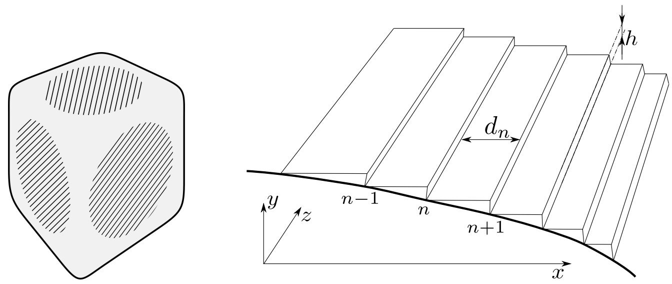
A solenoid of length $\ell = 20\,\text{cm}$ is wound around a vertical, cylindrical test tube made of glass, filled with water. The solenoid is thermally insulated from the water. The height of the water level is approximately 20 cm above the upper end of the solenoid, the diameter of the test tube is 1 cm, the number of turns of the coil is $N = 6000$. The atmospheric pressure is $p_0 = 101\,\text{kPa}$, the temperature of water is 293 K. Magnetic susceptibility of water is $\chi \equiv \mu_r - 1 = -9.04 \times 10^{-6}$. Vacuum permeability $\mu_0 = 12.57 \times 10^{-7}$ H/m.

The current in the solenoid is slowly increased until the water starts boiling inside the coil. At which current strength would this happen? Make reasonable approximations when needed. Note that the required current strength might be slightly too large for the present technology.

3 Staircase

The equilibrium shape of bodies in zero-gravity is determined by the minimum of their surface energy. Thus, for example, the equilibrium shape of a water droplet turns out to be spherical: the sphere has the smallest surface area among bodies of the same volume.

At low temperature, the equilibrium shape of crystals may have flat facets. The parts of the crystal surface that have a small angle φ with the facet are in fact staircases of rare steps on this facet. The height of such steps is equal to the period of the crystal lattice h.



Equilibrium surface profile $y(x)$ of a certain crystal and the corresponding microscopic staircase are shown schematically in the figure, where n denotes the step number, counting from $x = 0$. The profile shape at $x > 0$ can be approximated as $y(x) = -(x/\lambda)^{3/2}h$, where $\lambda = 45\,\mu\text{m}$ and $h = 0.3\,\text{nm}$.

A. Express the distance d_n between two adjacent steps as a function of n for $n \gg 1$.

B. The interaction energy E of two steps depends on the distance d between them as

$$E(d) = \mu\, d^{\nu},$$

where μ is a constant. Assume that only adjacent steps interact. Find the numerical value for the exponent ν.